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from which the term by term integrability of $S(x)$ follows when a and b are finite. The above inequality has, however, no sense when $b = \infty$, so that the term by term integrability in an infinite interval demands further investigation. If one does not want to invoke general theorems in this direction, such as are given, for instance, in Bromwich's *Infinite Series* or de la Vallee Poussin's *Cour d'Analyse,* one may proceed as follows in this problem.

Let

$$S(x) = \sum_1^\infty \frac{1}{n^4 + x^2}$$

be uniformly convergent for $x \geqq 0$; then the term by term integration in the finite interval $0 \leqq x \leqq a$ is legitimate:

$$\int_0^a S(x)dx = \sum_1^\infty \frac{1}{n^2} \arctan \frac{a}{n^2}.$$

Now arctan $(a/n^2) < (\pi/2)$ and all the terms of the right-hand series are positive; hence, for any $N \geqq 1$

$$\sum_1^N \frac{1}{n^2} \arctan \frac{a}{n^2} < \int_0^a S(x)dx < \frac{\pi}{2}\sum_1^\infty \frac{1}{n^2}.$$

Letting a tend toward infinity, *while keeping N fixed,* we find

$$\frac{\pi}{2}\sum_1^N \frac{1}{n^2} \leqq \int_0^\infty S(x)dx \leqq \frac{\pi}{2}\sum_1^\infty \frac{1}{n^2},$$

and finally letting N tend to infinity,

$$\int_0^\infty S(x)dx = \frac{\pi}{2}\sum_1^\infty \frac{1}{n^2} = \frac{\pi^3}{12}.$$

This point, that great caution should be exercised when inverting summation and integration in an infinite integration interval, or when the integrands may become infinite between the finite limits of integration, appears to me to be all the more important as the treatment of such questions in the current calculus textbooks is exceedingly inadequate.

393. Proposed by LAENAS G. WELD, Pullman, Ill.

Find the area of the least ellipse which can be drawn upon the face of a brick wall so as to inclose four bricks.

II. Solution by Frank R. Morris, Glendale, Calif.

In this solution the bricks are considered laid as in practical work.

Consider two of the bricks placed end to end in the same layer, a third in the layer above with its middle at the joint of the first two, and the fourth similarly placed in the layer below. Select as the origin the mid-point of the joint between the two bricks in the same layer and choose the x-axis parallel to the layers. Let the equation of the desired ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Since the four quadrants are symmetrical, it will be sufficient to consider the first one only. Call the upper, right-hand corner of the uppermost brick A and the corresponding corner of right-hand brick of the middle layer B. Let m represent the length of a brick and n its thickness. The ellipse will obviously pass through A or B or both. If it passes through A, whose coördinates are $(m/2, 3n/2)$, the equation is

$$\frac{\left(\frac{m}{2}\right)^2}{a^2} + \frac{\left(\frac{3n}{2}\right)^2}{b^2} = 1, \quad \text{or} \quad \frac{m^2}{a^2} + \frac{9n^2}{b^2} = 4.$$

Whence,

$$b = \frac{3an}{\sqrt{4a^2 - m^2}}, \quad \text{and the area } \pi ab \text{ is} \quad \frac{3\pi a^2 n}{\sqrt{4a^2 - m^2}},$$

which is a function, $f(a)$, of the independent variable a. Differentiating, and setting the derivative equal to 0, we have

$$f'(a) = \frac{6\pi a(2a^2 - m^2)}{(4a^2 - m^2)^{3/2}} = 0.$$

Whence $a = 0$ or $m/\sqrt{2}$.

$f'(a)$ increases continuously as a increases beyond $m/\sqrt{2}$. Hence, the least ellipse which will inclose the four bricks will be the one in which a is as near $m/\sqrt{2}$ as possible. The smallest value of a is given when the ellipse passes through the point B, this value being greater than $m/\sqrt{2}$.

It is perhaps evident that the least ellipse passing through B would pass through A also. It could be proved in the same manner as the first case.

The equation of the ellipse through B is

$$\frac{m^2}{a^2} + \frac{\left(\frac{n}{2}\right)^2}{b^2} = 1 \quad \text{or} \quad \frac{4m^2}{a^2} + \frac{n^2}{b^2} = 4.$$

Solving this simultaneously with the equation derived by substituting the coördinates of A for a and b we have

$$a = \frac{1}{4}\sqrt{\frac{35}{2}}\,m, \qquad b = \frac{1}{2}\sqrt{\frac{35}{3}}\,n.$$

Hence the area πab is

$$\frac{35\pi\sqrt{6}}{48}\,mn.$$

If allowance is made for mortar on all sides of each brick the solution is unchanged except m and n are increased by the thickness of the mortar. If the allowance is made at the joints of the four bricks but not at the points A and B, these points are $((m + t)/2,\ (3n + 2t)/2)$ and $((2m + t/2),\ n/2)$ where t is the thickness of the mortar.

MECHANICS.

317. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Show that the maximum area contained between the path of a projectile and the horizontal line is $v^2\sqrt{3}/8g^2$, where v is the velocity of projection.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

If the particle is projected in a direction making an angle α with the horizontal, the equations of motion are, t being the time,

$$x = v\cos\alpha\cdot t \quad \text{and} \quad y = v\sin\alpha\cdot t - \tfrac{1}{2}gt^2.$$

Equating y to zero, we find the path cuts the horizontal line through the point of projection when $t = 0$ and $t = \dfrac{2v\sin\alpha}{g}$. The area is given by $\displaystyle\int_{t=0}^{t=\frac{2v\sin\alpha}{g}} y\,dx = \frac{2v^4\cos\alpha\sin^3\alpha}{3g^2}$, which is to be a maximum. Setting the first derivative with respect to α equal to 0, we find $\alpha = 60°$. The maximum value of the area is then $= \dfrac{v^4\sqrt{3}}{8g^2}$ (v^2 being incorrect as given in the problem).

Also solved by J. L. RILEY, W. C. EELLS, THEODORE HOWARD, PAUL CAPRON, H. L. AGARD, HORACE OLSON, H. S. UHLER, and ELMER SCHUYLER.

318. Proposed by C. N. SCHMALL, New York City.

Given an inclined plane making an angle φ with the horizontal. A perfectly elastic ball is projected upward at an angle ψ with the inclined plane, so as to ascend it by bounds. Show that as the ball rebounds for the nth time, the angle of inclination of its path to the plane is

$$\tan^{-1}\left(\frac{\tan\psi}{1 - 2n\tan\varphi\tan\psi}\right)$$